Note 03 | 29 April 2010

A note on the very notion, „digital image“

The term *digital image* appears prominently in the title of our course. It also shows up in titles of books or conferences. We must know what this is: a digital image. At least we must try. Before attempting to formulate an answer, we should radically ask: *Is there anything like a digital image?*

A digital image, very directly, is an image that is digital. We may call a phenomenon "digital" when it appears as built up from *discrete* parts or elements, in units clearly separate from each other. But beware: each of those separate elements, units, entities as such may extend in space. If it does, taken as an entity it is not digital. It is analog. Only units which extend in space and/or time, can be sensually perceived. The digital, however, cannot be sensually perceived. It is a mental concept, not a sensual experience. This is important.

Take, as an example, the alphabet. An alphabet is a (usually finite) set of discrete elements, say $V = \{v_1, v_2, ..., v_n\}$. This notation indicates that we call the alphabet itself "V". Whatever its concrete elements may be, here we call them v_1, v_2, etc. So each "v" is the name of one element of V. The v's are brought into an order when we list them as members of the set. Therefore they are numbered by indices: 1, 2, etc.

The Latin alphabet is such a case:

$$V = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$$

The small letters indicate the actual shape of the elements of V, although we know that each of these letters may look quite different depending on the font we use. So the name "a" stands for a large class of all the shapes and sizes of printed letters "a". Let alone the "a"s of handwriting.

Get the idea? Calling a phenomenon "digital" means that we consider it from a rather abstract viewpoint. Images, taken as perceivable objects, are not digital. We can only think of them in digital terms: we assume a planar rectangle divided up into (small) units. Each of these units occupies a continuous area. A simple case is to take those units as squares of equal size. If there are m rows and n columns of such squares, the image is divided up into the following set:

$$I(m,n) = \{q_{ij} \mid i = 0,1,..., m-1, j = 0,1,..., n-1\}$$

and q_{ij} is a rectangle of the following kind:

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$$upper\_left\_corner(q_{ij}) = (i \cdot a, \quad j \cdot a)$$
$$size(q_{ij}) = a$$

But these rectangles describe only the geometry of the image. Its visibility needs assignment of a color to each of the small rectangles. Those rectangles are, as you have noted, the famous pixels. We may define a pixel as

$$p_{ij} = (q_{ij}, c_{ij})$$

The pixel here appears as a rectangle of size $a \times a$ at a specific location together with a color.

We may further formalize this:

$$q_{ij} = \left\{(x, y) \in \Re_2 \mid j \cdot a \leq x < (j+1)a, i \cdot a \leq y < (i+1)a\right\}$$
$$c_{ij} = (r_{ij}, g_{ij}, b_{ij}) \quad \text{with} \quad r_{ij}, g_{ij}, b_{ij} \in [0, 255].$$

The coordinate system of x and y has its origin in the upper left corner. Its x-axis extends to the right, its y-axis points downwards. This happens to be one of the arbitrary conventions of the trade.

More intuitively: a digital image is a finite set of pixels, arranged as a rectangle. A pixel is a pair of a location (and area) and a color-value. It is comfortable to think of a digital image as a mapping from a discretized rectangle into a color space. Again a bit of formalism ("ζ" is used here as the name of the image):

$$\zeta : R \rightarrow C$$

Here $R \subset \Re_2$ is the discretized rectangle, $C \subset [0, 255]^3$ is the color space of discrete colors, each of which consists of three components, and components happen to have 256 possible values, and

$$\zeta(q_{ij}) = c_{ij} \quad \text{for} \quad i \in [0, m-1], j \in [0, \ n-1].$$

Some remarks by students in reply to the question "What do you associate with the term “digital image?" were:

A representation of something in visible form.

How to display something on a screen.

A finite combination of pixels, where pixels are the language of displays.

In the discussion, we agreed that pixels could be viewed as the *elements* of a language of displays. The language itself consists of "statements" whose elements are the pixels. A statement would, in this analysis, be a set of pixels of any shape.

A *visible* image can be called a *digital image* when we transform it into the data structure of a finite pixel-matrix.